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Direct method for determination of shallow foundation settlements

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Preliminary report

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Direct method for determination of shallow foundation settlements

A direct procedure for determining settlement of shallow foundations, combining the modified hyperbolic function for nonlinear stress and strain ratio with correlations from penetration test results, is presented in the paper. The 10% load, i.e. 1% of settlement to equivalent foundations diameter ratio, is used in correlations. Laboratory tests are not needed in this novel procedure, which is a considerable advantage for coarse-grained soils. A very good correspondence was established between the load-based settlement curve calculated in this way, and the settlements measured in sand during load testing of five square foundations of variable size.

Key words:
shallow foundations, settlement, soil stiffness, maximum shear modulus, load testing

Prethodno priopćenje

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Direktan postupak za određivanje slijeganja plitkih temelja

Prikazan je direktan postupak za određivanje slijeganja plitkih temelja, koji kombinira modificiranu hiperbolnu funkciju za nelinearan odnos naprezanja i deformacija te korelacije iz rezultata penetracijskih pokusa. U korelacijama se koristi opterećenje za 10%, odnosno 1% omjera slijeganja i ekvivalentnog promjera temelja. Za ovaj novi postupak nisu potrebni laboratorijski pokusi, što je velika prednost za krupnozrna tla. Pokazuje se za ovako izračunane krivulje slijeganja u ovisnosti o opterećenju da je vrlo dobro poklapanje, s izmjerenim slijeganjima tijekom probnih opterećenja pet kvadratnih temelja različitih dimenzija na pijesku.

Ključne riječi:
plitki temelji, slijeganje, krutost tla, najveći posmični modul, probno opterećenje

Vorherige Mitteilung

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Direktes Verfahren zur Ermittlung der Setzungen von Flachfundationen

Das direkte Verfahren zur Ermittlung der Setzungen von Flachfundationen, das die modifizierte Hyperbelfunktion für das nichtlineare Verhältnis zwischen Spannungen und Dehnungen sowie die Korrelation der Resultate aus Penetrationsversuchen kombiniert, wird beschrieben. Bei den Korrelationen werden Lasten von 10%, bzw. 1%, des Verhältnisses der Setzungen zum äquivalenten Fundationsdurchmesser angesetzt. Für dieses neue Verfahren sind keine Laborversuche notwendig. Für die berechneten Setzungskurven in Abhängigkeit von den Lasten zeigt sich eine sehr gute Übereinstimmung mit gemessenen Setzungen bei Versuchslasten für fünf auf Sand liegende quadratische Fundationen verschiedener Abmessungen.

Schlüsselwörter:
Flachfundationen, Setzungen, Bodensteifigkeit, größtes Schubmodul, Versuchslast

1. Introduction

The determination of settlement of shallow foundations on coarse grained soils is still a subject of extensive research. The expression commonly used for the calculation of settlements is the one for shallow foundations on a linear elastic soil with the equivalent (or relevant) Young's modulus. However, soils are not linear elastic, but rather nonlinear elastoplastic. This *inter alia* leads to the problem of determination of the equivalent Young's modulus for the best prediction of soil settlement.

Load tests in the scope of which shallow foundations are gradually loaded in situ, and settlement measurements are taken, form a very precious basis for acquiring knowledge on the nonlinear soil behaviour (e.g. Briaud and Gibbens [1]). Such nonlinear load – settlement curves can not be obtained according to the theory of elasticity, regardless of the way in which the Young's modulus is varied.

Due to this fact, and also due to their simplicity, the use of correlations, with in situ penetration test results for calculating settlement of shallow foundations, is still prevailing in practice. This is especially the case when the foundation soil is coarse grained, because the acquisition of such undisturbed soil samples for laboratory testing is not common in standard practice. However, these correlations are not fit for nonlinear stress – strain relationships.

Numerical methods relating to the nonlinear continuum mechanics, such as the Finite Element Method, can be used for calculating settlement of shallow foundations using in situ and laboratory tests for determining parameters that describe the nonlinear stress – strain relationship. The problem with this approach is the acquisition of undisturbed samples of coarse grained soils, and performing complex laboratory tests on such samples, both of which are required for the determination of these parameters, which are numerous for an advanced representation of constitutive relationships.

Due to the mentioned problems, the prediction of shallow foundation settlements is still insufficiently reliable, as reported by Briaud and Gibbens [1] who present load test results for five quadratic footings on sand, with dimensions ranging from 1m to 3m. They organised a worldwide competition among practitioners and researchers for predicting settlements of the five footings prior to load tests and made all test results available. Various methods, including numerical modelling, were used in these predictions, but the results were close enough to measurements in rare instances only. After the measured settlements were published, they were considered as a basis for development of new methods for determining settlement of shallow foundations, but satisfactory results have not as yet been obtained.

Regarding laboratory tests required for predicting settlement of shallow foundations, triaxial and torsional shear tests have shown a significantly nonlinear relationship between deviatoric stress $q = \sigma_1' - \sigma_3'$ and vertical strain, or between shear stress and shear strain. Kondner [2] was the first to propose a hyperbolic function, using the mean effective stress $p' = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{3}$, to describe this nonlinear relationship. Then, Hardin and Drnevich [3] introduced a reference shear strain, which eliminates the dependence of hyperbola on p'. Fahey and Carter [4] modified Kondner's hyperbolic function by introducing a unique nonlinear relationship between the normalised secant shear modulus G/G_0, where G_0 is the maximum shear modulus at very small strains, and the normalised shear stress τ/τ_f, where τ_f is the shear strength of soil. This relationship has two constants, f and g.

It is well known that soils exhibit a linear elastic behaviour at very small strains (e.g. Burland [5]; Szavits-Nossan V. et al. [6]). In this range of strains, the Young's modulus is E_0, and the shear modulus is G_0. The relatively large soil stiffness at very small strains, which is encountered when measuring shear wave velocities in soil, can explain the significantly overestimated settlements which result from calculations based on traditional laboratory tests. These tests do not use devices for measuring soil stiffness at very small shear strains, which is nowadays possible by using local measuring devices. Very small shear strains are in the range of 10^{-5} to 10^{-4} (e.g. Lee et al. [7]). When soil is loaded to greater shear strains, the secant shear modulus nonlinearly decreases with an increase in shear strain by ten or more times with respect to G_0.

Mayne and Poulos [8] have proposed an expression for soil settlement under the centre of a flexible circular footing on a Gibson type of soil [9]. This is a nonhomogeneous, isotropic, elastic soil, having Young's modulus E_0 at the footing base, and the Young's secant modulus linearly increasing with the foundation soil depth down to the bedrock. Mayne [10] changed E_0 in this expression with a special case of the Fahey and Carter's [4] modified hyperbola (f = 1, g = 0.3) to describe reduction of the Young's modulus with an increase in deviatoric stress. Furthermore, instead of using the ratio q/q_f (deviatoric stress over deviatoric stress at failure), he uses the ratio p/p_f, where p is the uniform pressure on the footing and p_f is the bearing capacity of soil. Consequently, this new expression requires determination of the bearing capacity of soil.

A direct method for calculating settlement of shallow foundations on coarse grained and stiff fine grained soils is proposed in this paper based on the Mayne [10] approach, without the need to determine bearing capacity of soil. Instead of using a special case of the Fahey and Carter's [4] modified hyperbola, both parameters, f and g, are used. They are determined explicitly from correlations between the pressure required for 10 % and 1 % of the ratio of settlement and the equivalent diameter of a circular footing, and the results of in situ penetration tests [1, 17]. This enables determination of the two required parameters without resorting to laboratory shear tests.

2. Hyperbolic stress – strain relationships

The nonlinear stress – strain relationship for sand, which results from laboratory triaxial tests on soil samples, was first presented via a hyperbolic function by Kondner [2] in 1963. His function has the form

$$\sigma_1' - \sigma_3' = \frac{\varepsilon}{a + b\varepsilon} \tag{1}$$

where σ_1' and σ_3' are the major and minor principal effective stresses in the triaxial test, respectively, ε is the axial strain of the soil sample, and *a* and *b* are constants shown in Figure 1a. The ratio $1/a$ represents the initial, largest Young's modulus E_0 for the hyperbolic function, $1/b$ represents the hyperbola asymptote, i.e. the value of the limit stress difference of the hyperbola, $(\sigma_1' - \sigma_3')_{ult}$, which is different from the shear strength, or the stress difference at failure, $(\sigma_1' - \sigma_3')_f$, i.e. according to [2]

$$\left(\sigma_1' - \sigma_3'\right)_{ult} = \lim_{\varepsilon \to \infty}\left(\sigma_1' - \sigma_3'\right) = \frac{1}{b} = c\left(\sigma_1' - \sigma_3'\right)_f \tag{2}$$

where *c* is the ratio of the limit stress difference and the stress difference at failure. From expression (2), it follows that it takes an infinite strain to reach the limit stress difference, whereas failure occurs at a finite strain. The hyperbola is shown as a full line up to $(\sigma_1' - \sigma_3')_f$, in Figure 1.a, i.e. up to the strain ε_f and then it is shown as a dotted line. In a triaxial test, the soil sample follows the full line at $(\sigma_1' - \sigma_3')_f$, after reaching ε_f.
If function (1) is plotted so that the ordinate is $\frac{\varepsilon}{\sigma_1' - \sigma_3'}$, the hyperbola turns into a line, as shown by the red full line up to ε_f, and dotted line after that in Figure 1b. The slope of the blue line is 1:bc and it corresponds to $(\sigma_1' - \sigma_3')_f$. In a triaxial test, the soil sample would follow the hyperbola (straight line with the slope 1:*b*) up to ε_f, and the straight line with the slope 1:*bc* after that (full red and blue lines in Figure 1.b). Duncan and Chang [11] use parameter R_f instead of Konder's [2] parameter *c*, so that

$$R_f = \frac{1}{c} = \frac{\left(\sigma_1' - \sigma_3'\right)_f}{\left(\sigma_1' - \sigma_3'\right)_{ult}} \tag{3}$$

and they note that the value of R_f is between 0.75 and 1.00. The function (1) can now be written as

$$\sigma_1' - \sigma_3' = \frac{\varepsilon}{\frac{1}{E_0} + \frac{\varepsilon R_f}{\left(\sigma_1' - \sigma_3'\right)_f}} \tag{4}$$

odnosno

$$E = E_0\left(1 - R_f \frac{\sigma_1' - \sigma_3'}{\left(\sigma_1' - \sigma_3'\right)_f}\right) \tag{5}$$

where *E* is the Young's secant modulus.
Hardin and Drnevich [3] use function (1) with the shear stress and shear strain in the form

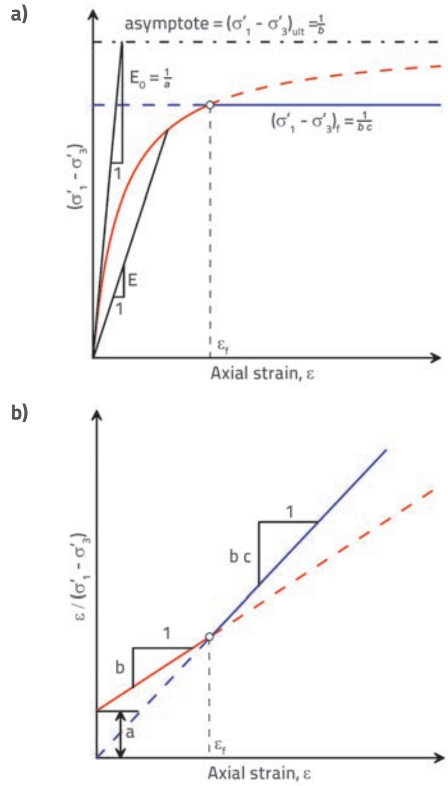

Figure 1. a) Hyperbolic function with corresponding constants, Young's secant modulus E, and deviatoric stress at failure; b) transformed hyperbolic function (red line) and state at failure (blue line)

$$\tau = \frac{\gamma}{\frac{1}{G_0} + \frac{\gamma}{\tau_{ult}}} \tag{6}$$

where τ is the shear stress, γ is the shear strain, G_0 is the initial shear modulus, and τ_{ult} is the limit shear stress on the hyperbola asymptote, and $G_0 = \frac{E_0}{2(1+\nu)}$, where ν is the Poisson's ratio.
Expression (6), just like expression (1) depends on the mean effective stress *p*', so that different values of *p*' give different curves. For example, according to Fahey [12], the initial shear modulus G_0 is proportional to $(p')^n$, where the value of exponent *n* ranges between 0.4 and 0.5, and the shear strength τ_f is proportional to *p*', in line with the Mohr-Coulomb principle. Hardin and Dmevich [3] suggest the expression that does not depend on *p*' by introducing the reference shear stress (Figure 2)

$$\gamma_r = \frac{\tau_{ult}}{G_0} \tag{7}$$

The coordinates of the point in which the initial hyperbola tangent cuts the asymptote are (γ_r, τ_{ult}).By introducing the secant shear modulus G, expression (6) becomes

$$\frac{G}{G_0} = \frac{1}{1 + \frac{\gamma}{\gamma_r}} \tag{8}$$

If it is assumed that the parameter R_f from expression (3) equals 1.00, Fahey [12] shows that expression (8) can be transformed into

$$\frac{G}{G_0}=1-\frac{\tau}{\tau_f} \tag{9}$$

Expression (9) represents a linear relationship, which is opposed to the torsional shear test results for Toyoura sand, which show a nonlinear relationship between the normalized shear modulus and the normalized shear stress, as shown in Figure 3. These results are presented by Teachavorasinskun et al. [13], and quoted by Lee et al. [7].

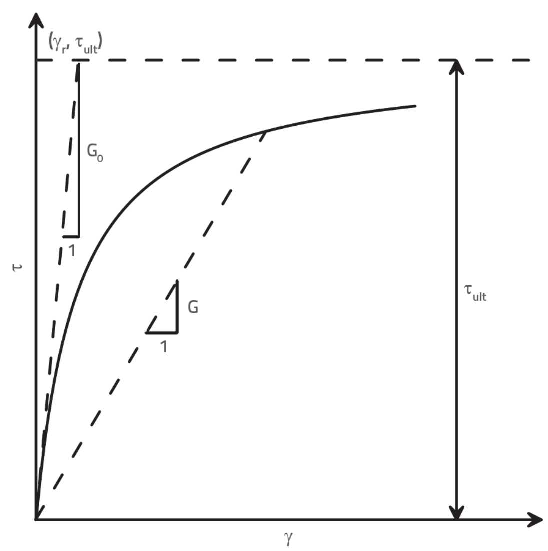

Figure 2. Hyperbolic function and reference shear stress (adapted from [3]

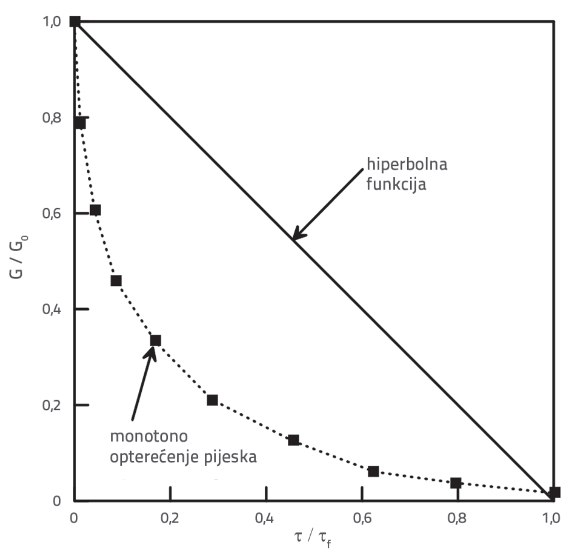

Figure 3. Hyperbolic function and torsional shear test results (according to [7], quoted by [13]

Results of many laboratory tests show similarities with results of the monotonous loading of sand from Figure 3. Mayne [14] collected the drained and undrained triaxial and torsional shear test results for eight sand samples, one clayey sand sample, and eight clay samples. The range of these results is shown in Figure 4, where it is significant to note that similar results are obtained for sand and clay.

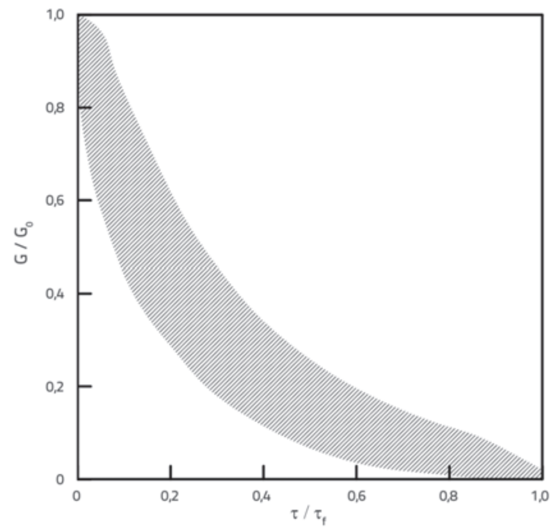

Figure 4. Range of triaxial and torsional shear test results for 17 sand and clay samples, adapted from [14]

Fahey and Carter [4] suggest an expression that represents a nonlinear relationship between the normalized shear modulus and normalized shear stress, without restrictions on the value of R_f from expression (3), in the form

$$\frac{G}{G_0}=1-f\left(\frac{\tau}{\tau_f}\right)^g \tag{10}$$

where f and g are parameters of the model. Parameter f is a substitute for R_f and parameter g dictates the shape of the nonlinear curve. These two parameters should be determined from the laboratory triaxial or torsional shear tests.

Figure 5 shows curves from equation (10) for various values of parameters f and g. It also shows the monotonous sand loading curve from Figure 3. Fahey and Carter [4] suggest values of f = 0.98 and g = 0.25 for the curve that best fits the sand loading results. Mayne [14] takes f = 1, and various values for parameter g. For f = 1, and g = 1 equation (10) gives a line, as a standard hyperbolic curve, which is also shown in Figure 3. Equation (10) can also be written as

$$\frac{E}{E_0}=1-f\left(\frac{q}{q_f}\right)^g \tag{11}$$

where E is the secant modulus of elasticity, q is the deviatoric stress, and q_f is the deviatoric stress at failure.

Equation (11), as a modified hyperbolic function, will serve in this paper for development of the new direct method for determining shallow foundation settlements in coarse-grained and fine-grained stiff soils.

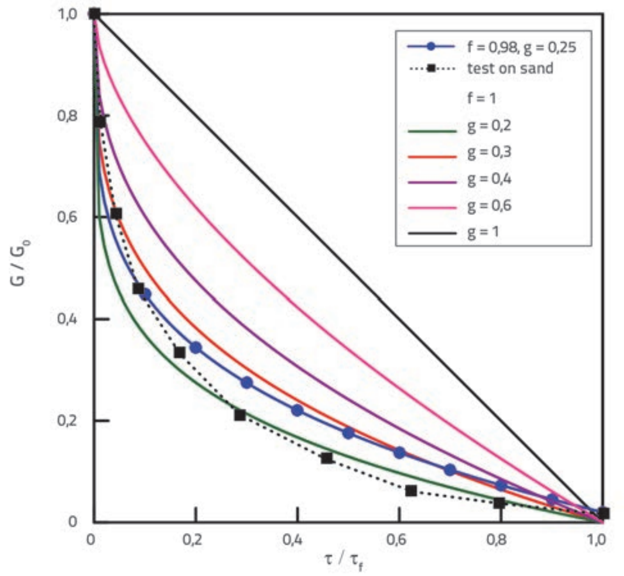

Figure 5. Monotonous loading of sand from Figure 3; *f* = 0.98 and *g* = 0.25 adapted from [4]; curves for f =1, adapted from [14]

3. Shallow foundation settlements according to theory of elasticity

The settlement of shallow foundations loaded axially by a force smaller than the soil bearing capacity is in practice often determined using the theory of elasticity (e.g. according to Eurocode 7) by

$$s = \frac{pBI}{E_m} = \frac{p}{k} \tag{12}$$

where p is the average contact pressure between the foundation and the soil, B is the foundation width, I is the coefficient of settlement that is dependent on the foundation shape, its embedment depth, and the soil layer thickness, and E_m is the corresponding (equivalent) Young's modulus that will, according to expression (12), give an approximate settlement value for soil that is not linearly elastic. The parameter k

$$k = \frac{p}{s} = \frac{E_m}{BI} \tag{13}$$

determines the stiffness of the foundation soil. This stiffness is sometimes referred to as the Winkler coefficient of the subsoil. Davis and Poulos [15] commented on the validity of applying the expression from the theory of elasticity, with an appropriate choice of the equivalent Young's modulus, for the determination of settlement of shallow foundations. They state that the theory of elasticity can be applied in practice, provided that laboratory determination of the equivalent Young's modulus is carried within the range of stresses relevant for field conditions.

However, there is still a problem with laboratory tests on undisturbed coarse grained samples.

Numerous expressions for calculating displacement of elastic bodies subjected to loading on their surface or in their interior have been deduced from the theory of elasticity. Poulos and Davis [16] have selected those that might be useful in geotechnical practice. The simplest such case is the one of a circular plane under uniform normal pressure on the surface of the isotropic and homogeneous elastic half space. For a flexible circular footing having the diameter d on this kind of soil with Young's modulus E, and Poisson's ratio ν, the foundation soil stiffness for calculating the footing settlement at its centre is given in the following expression (Timoshenko and Goodier [17])

$$k = \frac{E}{\left(1-\nu^2\right)d} \tag{14}$$

where the coefficient of settlement is

$$I = 1 - \nu^2 \tag{15}$$

In case of a rigid circular foundation, the rigidity k is given by

$$k = \frac{4}{\pi}\frac{E}{\left(1-\nu^2\right)d} \tag{16}$$

Mayne and Poulos [8] suggested an approximate expression for a much more general case of non-homogenous, isotropic, linearly elastic soil, where Young's modulus linearly increases with the soil depth z, so that it is equal to E_0 at the footing base, and then it increases with the coefficient k_E, so that $E = E_0 + k_E z$ (Figure 6). The soil with these characteristics is called the Gibson type of soil [9]. A flexible circular footing of diameter d, thickness t, Young's modulus E_f, and a constant Poisson's ratio ν, is embedded to the depth D_f in the soil of thickness h, measured from the footing base down to the bedrock.

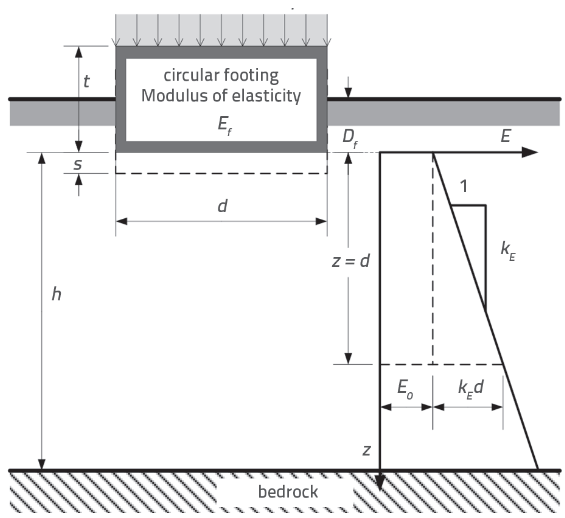

Figure 6. Notations related with expression (17) for settlement of circular footing embedded in Gibson type of soil [9]

According to Mayne and Poulos [8], the settlement under the centre of a flexible circular footing on the Gibson type of soil is given by

$$s = \frac{pdI_G I_F I_E \left(1-\nu^2\right)}{E_0} = \frac{p}{k} \tag{17}$$

so that the stiffness of the soil is given by

$$k = \frac{E_0}{dI_G I_F I_E \left(1-\nu^2\right)} \tag{18}$$

where I_G, I_F and I_E are the factors that will be defined below. According to the same authors, the expression (17) can also be used, with a minor error, for a rectangular footing of area A, the width and length of which not differing greatly from one another, in such a way that an equivalent diameter of the footing is calculated as

$$d = 2\sqrt{\frac{A}{\pi}} \tag{19}$$

I_G is the factor of non-homogeneity, which depends on the parameter

$$\beta = \frac{E_0}{k_E d} \tag{20}$$

and the ratio *h/d*. Mayne and Poulos [8] show the dependence of I_G on β in the form of a diagram for various values of *h/d*. The approximate expression for the non-homogeneity factor can be written as

$$I_G \approx \frac{1.6\dfrac{h}{d}}{\left(1+\dfrac{0.6}{\beta^{0.8}}\right)\left(1+1.6\dfrac{h}{d}\right)} \tag{21}$$

I_F is the stiffness factor of the elastic footing, which depends on the footing thickness *t* and its Young's modulus. It is given by the expression

$$I_F = \frac{\pi}{4} + \frac{1}{\dfrac{1}{1-\dfrac{\pi}{4}} + 10\left(\dfrac{E_f}{E_0+\dfrac{1}{2}k_E d}\right)\left(\dfrac{2t}{d}\right)^3} \tag{22}$$

I_E is the factor of the footing embedment, given by

$$I_E \approx 1 - \frac{1}{3.5\exp\left(1.22\nu-0.4\right)\left(\dfrac{d}{D_f}+1.6\right)} \tag{23}$$

Mayne [10, 14] extends the expression (17) for settlement with the Fahey and Carter [4] function (11), using specific values of the two parameters, i.e. *f* = 1 and g = 0.3. Mayne claims that these values give reasonable approximations for determining settlement of shallow foundations. The extended expression takes into account reduction of the Young's secant modulus with an increase in strain, relative to its initial value E_0. Mayne also replaces in expression (11) the ratio of the deviatoric stress and the deviatoric stress at failure (q/q_f) with the ratio of (p/p_f), where *p* is the uniform pressure on the footing, and p_t is the bearing capacity of soil. The ratio (q/q_f), which is analogous to (τ/τ_f), equals 1/FS, where FS is the factor of safety, and the same can be applied to the ratio (p/p_f). Thus, according to [10, 14]

$$s = \frac{pdI_G I_F I_E \left(1-\nu^2\right)}{E_0\left(1-\left(\dfrac{p}{p_f}\right)^{0.3}\right)} \tag{24}$$

Mayne [10] uses the expression (24) to calculate settlements corresponding to a load test performed at A&M Texas University [1] on a 3 m square footing (North) on sand. Figure 7 shows settlements measured according to [1] and calculated according to (24), based on the parameters given in [14]. It should be noted that expression (24) gives positive values of settlement, whereas settlements are shown as negative values in Figure 7 (and in other load - settlement diagrams). The soil bearing capacity is calculated according to the Vesić method. Vertical portions of the curve with settlements measured in Figure 7 represent the soil creep during 30 minutes at constant load. Briaud and Garland [18] (cited in [1]) suggested the following expression for prediction of soil creep

$$\frac{s_1}{s_2} = \left(\frac{t_1}{t_2}\right)^n \tag{25}$$

where s_1 is the settlement after time of creep t_1, s_2 is the settlement after time of creep t_2 and *n* is the exponent related to soil viscosity. Typical values of the parameter *n* for sand range from 0.005 to 0.03 [1]. Briaud and Gibbens [1] used the expression (25) to estimate the creep of sand at the location of A&M University, Texas, with n = 0.03. If it is assumed in equation (25) that t_1 = 50 years, t_2 = 30 minutes, and *n* = 0.03, then s_1/s_2 = 1.50, which means that the settlement after 50 years will be by 50 % greater than the one after 30 minutes.

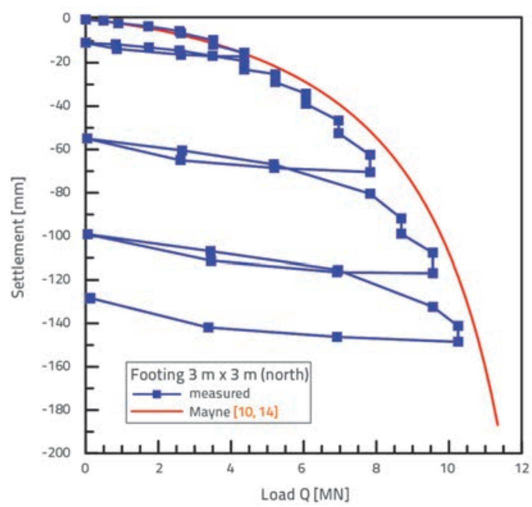

Figure 7. Measurements from load test (A&M Texas) [1] and settlements calculated from equation (24)

Despite the good correspondence between the measured and calculated settlements in Figure 7, it should be emphasized that Mayne [10, 14] showed this comparison for only one footing on sand. This approach should, therefore, be verified on a greater number of load tests performed on footings. In addition, expression (24) uses the soil bearing capacity, which can not be unambiguously determined by any of the methods that are often used in practice.

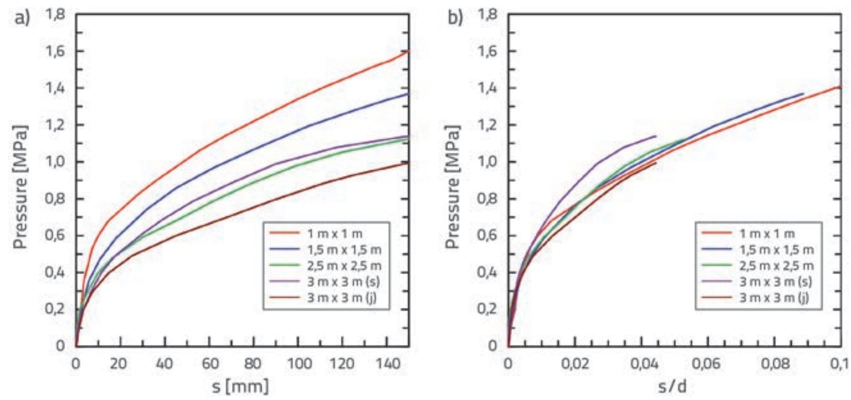

Figure 9. a) Pressure – settlement curves according to measured values from Figure 8; b) pressure – "strain" curves (adapted from [1])

4. New direct method for calculating settlement of shallow foundations

This new method uses equation (24), with both Fahey and Carter [4] parameters *f* and *g*, so that

$$s = \frac{p d I_G I_F I_E \left(1-\nu^2\right)}{E_0 \left(1 - f\left(\frac{p}{p_f}\right)^g\right)} \tag{26}$$

Briaud and Gibbens [1] present results from five load tests on square footings ranging in dimensions from 1 m to 3 m on sand, performed at the A&M University, Texas. They show that dividing settlement with the footing width (*s*/*B*), or its equivalent diameter (*s*/*d*), normalises pressure – "strain" curves so that these curves almost overlap, at least up to the "strain" of *s*/*B* = 0.05. Thus, the footing width has no effect on such curves up to this "strain". They also state that extrapolation of this finding up to soil failure would give the same bearing capacity for all footings at a given location [1]. This is contrary to the basic Terzaghi expression for the soil bearing capacity of strip foundation, $p_f = \frac{1}{2}\gamma B N_\gamma$, where g is the unit weight of soil, and N_γ is the bearing capacity factor depending on the angle of friction j'. Briaud and Gibbens [1] interpret this in two ways: either the soil bearing capacity does not depend on the foundation width, or the bearing capacity factor N_γ depends not solely of φ' but also on foundation width. In any case, this is an additional indication that it is not advisable to use the soil bearing capacity for settlement calculations.

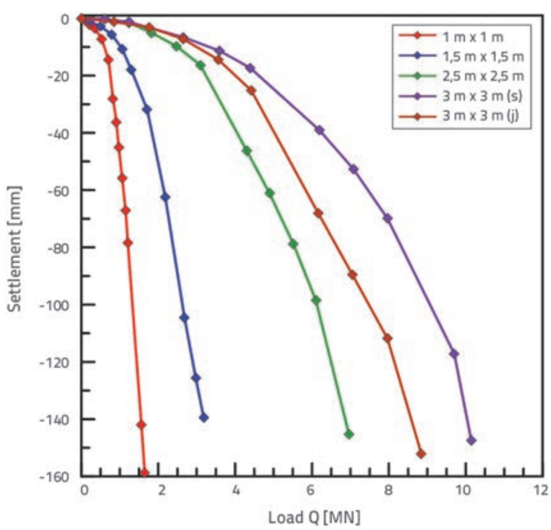

Figure 8. Load-settlement curves for five footings at A&M Texas; average settlements after 30 minutes of creep; (N) North, (S) South (adapted from [1])

Figure 8 shows load - settlement curves for all five foundations at A&M Texas. The curves present average settlement values after 30 minutes of creep. Figure 9.a shows the same results in the form of pressure - settlement curves. Figure 9.b shows curves similar to those given in Figure 9.a, with settlement being replaced by "strain" (*s/d*).

Briaud and Gibbens [1] use the "strain" (*s*/*d*) = 0.05 for the five footings, because all five of them were loaded to a settlement of 15 cm, and it can be seen from Figure 9b that all foundations reached this "strain". The authors recommend that the soil bearing capacity should be defined for the "strain of 10 %, i.e. for (*s*/*d*) = 0.1. Furthermore, they propose correlations for the pressure required to reach (*s*/*d*) = 0.1, with the penetration test results (SPT - standard penetration test; CPT- cone penetration test), as follows

$$p_f = p_{0.1} = \frac{N}{12} \quad N\,[\text{MPa}],\ N \text{ number of SPT blows/0,3 m} \tag{27a}$$

$$p_f = p_{0.1} = \frac{q_c}{4} \tag{27b}$$

where q_c is the cone tip resistance.

Since the allowed pressure can be calculated as 1/3 of the bearing capacity, authors [1] also give correlations for the allowed pressure p_d that roughly corresponds to *s*/*d* = 0.01 from Figure9.b, so that

$$p_a = p_{0.01} = \frac{N}{36} \quad N\,[\text{MPa}],\ N \text{ number of SPT blows/0,3 m} \tag{28a}$$

$$p_a = p_{0.01} = \frac{q_c}{12} \tag{28b}$$

Briaud and Gibbens [19] give slightly different figures for correlations

$$p_{0,1} = 0{,}075\ N\,[\text{MPa}],\ N \text{ number of SPT blows/0,3 m} \tag{29a}$$

$$p_{0,1} = 0{,}23\ q_c \tag{29b}$$

$$p_{0,1} = 1{,}7\ P_L \tag{29c}$$

where P_L is the pressuremeter limit pressure, and

$$p_{0,1} = 0{,}03\ N\,[\text{MPa}],\ N \text{ number of SPT blows/0,3 m} \tag{30a}$$

$$p_{0,1} = 0{,}03\ q_c \tag{30b}$$

$$p_{0,1} = 0{,}7\ P_L \tag{30c}$$

Equation (26) can now be written as

$$s = \frac{p d I_G I_F I_E \left(1-\nu^2\right)}{E_0 \left(1 - f\left(\dfrac{p}{p_{0.1}}\right)^g\right)} \tag{31}$$

where the soil bearing capacity no longer appears in the traditional sense, but rather as pressure $p_{0.1}$. This is a great advantage of this method, which allows for parameters f and g to be determined from the two equations that are used to correlate $p_{0.1}$ and $p_{0.01}$ with SPT or CPT results. This gives explicit expressions for f and g in the form

$$f = 1 - \frac{p_{0.1}}{0.1E_0} I_G I_F I_E \left(1-\nu^2\right) \tag{32}$$

$$g = \frac{\log\left[\dfrac{1}{f}\left(1 - \dfrac{p_{0.01}}{0.01E_0} I_G I_F I_E \left(1-\nu^2\right)\right)\right]}{\log \dfrac{p_{0.01}}{p_{0.1}}} \tag{33}$$

The use of the pressure $p_{0.1}$ instead of the soil bearing capacity which appears in Mayne's expression for settlement of shallow foundations [10, 14], and the explicit determination of parameters f and g from two correlations among expressions (27) to (30), instead of their determination in the laboratory, are the basic advantages of this new direct method for calculating settlement of shallow foundations.

5. Verification of new method

5.1. Description of test field and results of soil behaviour prediction during load tests

The test field that is used for verification of the new method for calculating settlement of shallow foundations is located at Riverside Campus of A&M Texas University, close to the College Station [18]. Three square footings, with dimensions 1 x 1 m, 1.5 x 1.5 m and 2.5 x 2.5 m, and two footings of 3 x 3 m were tested, three of them embedded 0.76 m (Table 1.) in a 11m thick layer of uniform, medium dense silicate silty sand. The lower boundary of this layer, in contact with a layer of stiff clay, is beyond the influence of loading on the soil. The groundwater is 4.9 m below the ground level. Among other measurements, vertical and horizontal displacements of footings were determined, and the load was applied every 30 minutes (in some cases even after 24 hours) until the settlement of 15 cm was reached. The soil at the test field was extensively tested by in-situ investigations and in the laboratory. Seismic cone penetration tests, with pore pressure measurements, provided continuous profiles of the velocity of shear waves, cone tip and shaft resistances, and the pore pressure along the depth of the sand layer. Using the shear wave velocity v_s, the maximum shear modulus can be determined from $G = \rho v_s^2$, where ρ is the soil density.

It is interesting to note that, prior to performing load tests at A&M Texas, a competition was announced in which the competitors were to predict the load at a settlement of 25 mm, the load at a settlement of 150 mm, and soil creep during 30 minute intervals between two load phases. The site documentation was requested by 150 candidates, and 31 contestants from 8 countries delivered the requested data.

None of the contestants predicted all 10 results (two required loads for each of 5 foundations) within ± 20 % of measured values. Two contestants met this criterion for 8 results. For loads at a settlement of 25 mm, 80 % of calculated settlements

Table 1. Dimensions of tested footings

Lenght x width [m x m]	Thickness [m]	Embedment depth [m]	Notation in the text [m]
0.991 x 0.991	1.168	0.711	1 x 1
1.505 x 1.492	1.219	0.762	1.5 x 1.5
2.489 x 2.496	1.219	0.762	2.5 x 2.5
3.004 x 3.004	1.219	0.762	3 x 3 sjever
3.023 x 3.016	1.346	0.889	3 x 3 jug

were greater than the measured ones, and for loads at a settlement of 150 mm, there were 63 % such results. These results indicate that shallow foundations could be designed more economically.
Eighteen contestants used the Schmertmann method [21] (cited in [1]), 9 competitors used the Burland and Burbidge method [22], and 8 competitors conducted numerical modelling with the finite element method (some contestants used more than one method). It is hard to say which one of these methods is the best, because contestants have combined one or more of these methods based on their previous experience.
Briaud and Gibbens [1] carried out independent calculations according to 12 methods for settlement prediction, and 6 methods for predicting bearing capacity of soil. They showed that the Schmertmann method [23] (cited in [1]) and the Peck and Bazaraa method [24] (cited in [1]) were the best, even though they both gave somewhat smaller settlements compared to the ones measured at the same load. The best methods that gave greater settlements than the measured ones at the same load, are the one by Briaud [25] (cited in [1]) and the one by Burland and Burbidge [22]. Briaud's simple 0.2 q_c method [26] (cited in [1]), proved to be the best for calculating bearing capacity of soil, whereas most other methods gave 25 % to 42 % smaller values compared to the ones obtained by measurements.

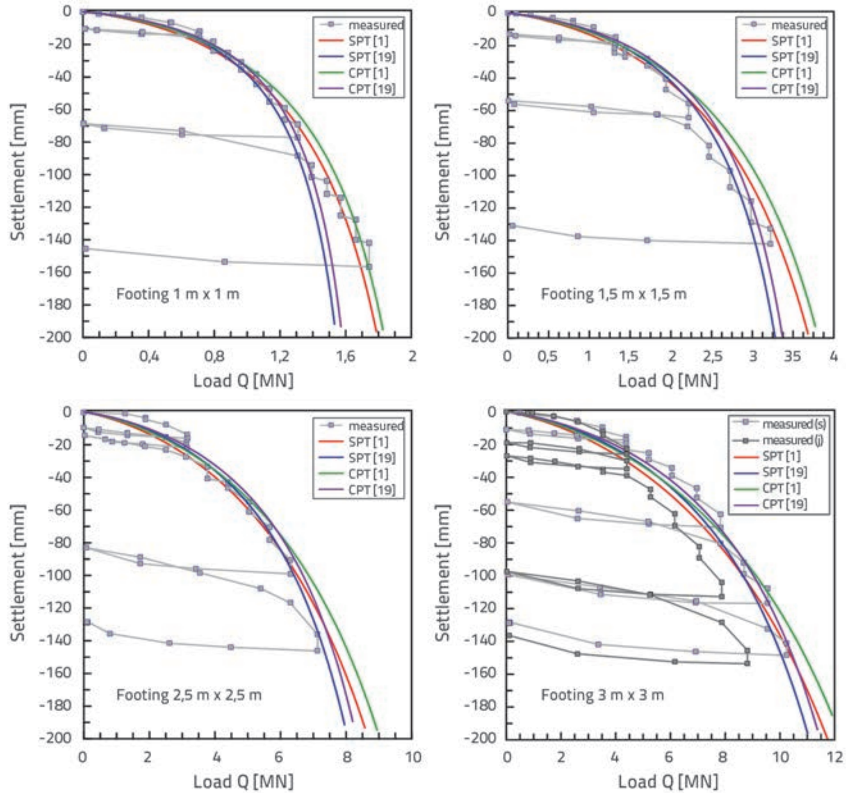

Figure 10. Measured settlements and calculated settlements using the new direct method with correlations from equations (27) to (30)

5.2. Simulations of A&M Texas load test results by new method

The new direct method was used for calculating the load - settlement relationship for all 5 A&M Texas footings, according to equation (31). The corresponding results are shown in Figure 10. The four curves shown in each diagram refer to the four types of correlations used to determine $p_{0.1}$ and $p_{0.01}$ from SPT blow count N [1] and [19], and from the cone tip resistance q_c [1] and [19], according to equations (27) to (30). The average number of SPT blows is N = 18.8, and the average cone tip resistance is q_c = 7 MPa. The values $p_{0.1}$ and $p_{0.01}$ were inserted in equations (32) and (33) in order to determine parameters f and g in equation (31). E_0 was determined from the average shear wave velocity v_s and Poisson's ratio ν = 0.2. Thus, E_0 was calculated as 230.4 MPa. Factors I_σ, I_p, I_E were determined for each footing from equations (21), (22) and (23) respectively, with k_E = 0, because Young's modulus did not change significantly throughout the sand layer, so that, according to (20), $1/\beta$ = 0.
It can not be seen from diagrams presented in Figure 10 which of the four correlations is the best. They all correspond very well to measured settlements, except for the footing 3m x 3m (South) at higher load values. It is, however, possible to single out the

Table 2. Measured and calculated values of Q_{25} and Q_{150} (kN)

Footing	Measured		Calculated			
	Q_{25}	Q_{150}	Q_{25}	%	Q_{150}	%
1 m x 1 m	850	1740	817	-3.9	1673	-3.9
1.5 m x 1.5 m	1500	3400	1484	-1.1	3370	-0.9
2.5 m x 2.5 m	3600	7100	3012	-16.3	7773	+9.5
3 m x 3 m (sjever)	5200	10250	3880	-25.4	10437	+1.8
3 m x 3 m (jug)	4500	9000	3907	-13.2	10520	+16.9

correlations with the SPT blow count [1] (red curves) when considering differences between the calculated and measured settlements for the whole range of loads, and for all footings. These curves enabled calculation of the loads Q_{25} and Q_{150} for achieving settlement values of 25mm and 150 mm, respectively, which was requested in the described competition. The measured and calculated values are presented in Table 2.
It can be seen from Table 2, that excellent results have been obtained. Nine out of ten forces are within ± 20 % from the measured data, and the tenth force is not much out of this range. Moreover, five results are within ± 5 %. Seven out of ten calculated settlements are greater than those measured at the same load.
This shows that the verification of the new direct method for calculation of shallow foundation settlements gives credibility to this method, although additional load tests have to be carried out for shallow foundations in sands and stiff clays.

6. Conclusion

A new direct method for calculating settlement of shallow foundations in sand and stiff clay is presented. This method is based on the Fahey and Carter [4] modified hyperbola for the nonlinear stress – strain relationship, and on the Mayne and Poulos [8] expression for settlements, where they used a linearly increasing Young's modulus with the soil depth according to the Gibson type of soil [9], and factors of non-homogeneity, stiffness of the elastic footing, and footing embedment. It is also based on the method presented by Mayne [10, 14], who introduced the bearing capacity of soil in the expression for settlement of shallow foundations, for a special case of modified hyperbola [4]. Finally, the method makes use of the Briaud and Gibbens [1, 19] correlations between the pressure required to reach the ratio of settlement and equivalent footing diameter of 10 %, and 1 %, and the results of in situ penetration tests. The shortcoming of the Mayne method [10, 14] is in using the soil bearing capacity, which cannot be determined unequivocally. This shortcoming is bypassed in the new method by replacing the soil bearing capacity with the pressure required to reach the ratio of settlement and the equivalent footing diameter of 10 %. It is suggested in [4] to determine values of parameters f and g by means of laboratory tests. This is the reason why Mayne [10, 14] uses a special case for these parameters (f = 1, g = 0.3), which are, again, the result of extensive laboratory testing. In the new method, the values of parameters f and g are obtained from explicit equations, by using correlations [1, 19], which is a great advantage of this method.
The verification of the new method was carried out using results of five load tests conducted at the A&M University, Texas, on square footings measuring 1m to 3m on sand. Settlements calculated by the new method correspond very well with the measured data for all footings. Calculation results show that loads required to reach a settlement of 25mm and a settlement of 150mm for each of five footings are within ± 20 % from measured values in 9 out of 10 cases, and in 5 cases they are within ± 5 %. These results are much better than 31 predictions made prior to load tests.
This new method can easily be applied in practice by using in situ penetration tests (SPT or CPT), and by measuring the shear wave velocity. The footing dimensions, embedment depth, and the soil layer thickness, have to be known, and no laboratory tests are necessary. It would be useful to carry out additional verifications of the new method by testing load for shallow foundations on sand and stiff clay.

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