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Advanced nonlocal hygrothermal buckling of FGM sandwich plates

Autori:



Assist.Prof. **Idder Abdelghani**, PhD. CE
Ahmed Draia University, Algeria
Faculty of Science and Technology
Department of Civil Engineering
abdelghani.idder@univ-adrar.edu.dz
Corresponding author



Assist.Prof. **Moulay Ali Abderrahmane**, PhD. CE
Ahmed Draia University, Algeria
Faculty of Science and Technology
Department of Civil Engineering
abd.moulayali@univ-adrar.edu.dz



Frih Ahmed, PhD. CE
Tahri Mohamed University, Algeria
Department of Civil Engineering and Hydraulics
ahmed_frih@outlook.com



Assist.Prof. **Abdelhak Aouadi**, PhD. CE
University of Bouira, Algeria
Faculty of Applied Sciences
Department of Civil Engineering
a.aouadi@univ-bouira.dz

Research Paper

Idder Abdelghani, Moulay Ali Abderrahmane, Frih Ahmed, Abdelhak Aouadi

Advanced nonlocal hygrothermal buckling of FGM sandwich plates

This study develops a refined trigonometric shear deformation theory combined with Eringen's nonlocal elasticity for the hygrothermal buckling analysis of functionally graded sandwich nanoscale plates on Winkler–Pasternak foundations. The formulation satisfies zero transverse shear stresses at the surfaces without correction factors. Analytical solutions are obtained using Navier's method for simply supported plates subjected to temperature and moisture gradients. Parametric studies assess the effects of geometric slenderness, foundation stiffness, and the shear function parameter q . The results agree closely with benchmark results of the trigonometric shear deformation theory (TSDT), with errors below 2%. The proposed approach enables efficient and accurate stability prediction of advanced sandwich structures, supporting the design of lightweight and durable components for civil, marine, and aerospace engineering applications.

Key words:

functionally graded sandwich plates, hygrothermal buckling, refined shear deformation theory, nonlocal elasticity, elastic foundation, Navier's method

Prethodno priopćenje

Idder Abdelghani, Moulay Ali Abderrahmane, Frih Ahmed, Abdelhak Aouadi

Napredno nelokalno higrotermičko izvijanje FGM sendvič ploča

Ovo istraživanje razvija unaprijedenu trigonometrijsku teoriju posmične deformacije u kombinaciji s Eringenovom nelokalnom elastičnošću za analizu higrotermičkog izvijanja funkcionalno gradiranih sendvič nanoskalnih ploča na Winkler–Pasternak elastičnim podlogama. Formulacija zadovoljava uvjet nultih poprečnih posmičnih naprezanja na površinama bez potrebe za korekcijskim faktorima. Analitička rješenja dobivena su primjenom Navierove metode za jednostavno oslonjene ploče izložene temperaturom i vlagom uzrokovanim gradijentima. Parametarska analiza ispituje utjecaj geometrijske vitkosti, krutosti podloge i parametra posmične funkcije q . Rezultati pokazuju vrlo dobro slaganje s referentnim rezultatima trigonometrijske teorije posmične deformacije (TSDT), s pogreškama manjim od 2%. Predloženi pristup omogućuje učinkovito i točno predviđanje stabilnosti naprednih sendvič konstrukcija, podupirući projektiranje laganih i dugotrajnih komponenti za primjene u građevinarstvu, pomorstvu i zrakoplovstvu.

Ključne riječi:

funkcionalno gradirane sendvič ploče, higrotermičko izvijanje, unaprijeđena teorija posmične deformacije, nelokalna elastičnost, elastična podloga, Navierova metoda

1. Introduction

Functionally graded material (FGM) sandwich plates represent a high-performance class of composite structures in which the material composition and properties vary continuously through the thickness [1]. This continuous gradation, typically achieved by transitioning between two or more constituent phases, commonly ceramics and metals, according to a prescribed volume fraction law (e.g. the power-law distribution) [2], enables the simultaneous optimisation of stiffness, strength, and thermal resistance within a single structural element [3]. Unlike conventional laminated sandwich panels, FGMs eliminate abrupt interfaces where sudden changes in elastic modulus, thermal expansion coefficient, or moisture diffusivity may cause stress discontinuities, interfacial debonding, and premature failure [4].

In civil engineering, these advanced plates are increasingly used in lightweight bridge decks [5], roof shells [6], marine platforms [7], and protective panels [8], where they must withstand coupled mechanical and environmental actions [9]. Hygrothermal environments, characterised by simultaneous temperature gradients and moisture variations, can induce complex deformation mechanisms [10]. Thermal gradients generate differential expansion or contraction, while moisture ingress alters stiffness and induces swelling strains [11]. Over time, cyclic hygrothermal loading can reduce stability, shift natural frequencies, and significantly decrease critical buckling loads, making accurate prediction essential for safe and durable design [12].

Support conditions further influence the structural response. Numerous practical systems involve interaction with elastic substructures, which can be realistically modelled using a three-parameter foundation: the Winkler modulus to represent normal stiffness [13–15], the Pasternak shear coefficient to account for transverse shear coupling [16, 17] and a damping parameter to simulate viscoelastic effects [18, 19]. These foundation parameters strongly affect load–deflection behaviour, critical buckling temperatures, and post-buckling performance, and their inclusion is essential in predictive models for real-world applications.

Over recent decades, the modelling of FGM sandwich plates has progressed from classical plate theory (CPT) [20] and first-order shear deformation theory (FSDT) [21] to higher-order shear deformation theories (HSDTs) [22] and equivalent single-layer (ESL) approaches [23]. HSDTs capture realistic through-thickness shear stress variations without the need for empirical shear correction factors [24–29], but they often require a large number of variables and complex governing equations, particularly in hygrothermoelastic problems on elastic foundations, thereby limiting their routine use in design practice. Numerical approaches such as the finite element method (FEM) [30] and the differential quadrature method (DQM) [31] can address these complexities, but at a high computational cost, especially in extensive parametric studies [32].

In engineering practice, functionally graded (FG) sandwich plates are increasingly employed in components such as façade panels, lightweight roofs, floor elements, and cladding systems, where variations in temperature and humidity can significantly affect their mechanical response. While existing plate models provide valuable tools for analysing such structures, there remains a need for formulations that consistently incorporate hygrothermal effects, material gradation, nonlocal behaviour, and the influence of elastic foundations. Accounting for elastic foundation effects enables a more realistic representation of the interaction between the plate and its supporting medium, such as soil, insulation layers, or substructures. This improves the prediction of deflections, vibrations, and overall stability. It is particularly beneficial in civil engineering applications, where considering foundation compliance can enhance the design of floor systems, bridge decks, and façade panels by limiting excessive deformations and ensuring both structural safety and serviceability under operational loads.

Recent studies have further advanced the modelling of FG and FGM sandwich nanoplates under coupled hygrothermal and mechanical effects. Benachi et al. [33] examined the elastic instability of functionally graded sandwich nanoplates under hygrothermo-mechanical loading using a quasi-3D five-variable shear deformation theory combined with Eringen's nonlocal elasticity and a two-parameter elastic foundation. Zaitoun et al. [34] investigated the hygrothermal buckling behaviour of FG sandwich plates resting on a visco-Pasternak foundation using a higher-order shear deformation theory with four unknowns and a viscoelastic damping term. More recently, Refrafi et al. [35] analysed the hygrothermo-mechanical buckling of FG sandwich plates on Winkler–Pasternak foundations using a shear deformation model that satisfies zero transverse shear stresses without requiring shear correction factors. While these works provide significant contributions, they typically rely on complex quasi-3D theories, integral terms, or predefined shear functions. In contrast, the present study employs a newly proposed trigonometric shape function with a tunable parameter q , integrates nonlocal elasticity within a unified hygrothermal formulation, and derives an efficient closed-form Navier-type solution for FG sandwich plates on Winkler–Pasternak foundations.

To address these challenges, the present study develops a refined four-variable trigonometric shear deformation theory (TSDT) integrated with Eringen's nonlocal elasticity within a unified hygrothermal formulation. The adopted shear function inherently satisfies traction-free transverse shear conditions, while the governing equations are derived using the principle of virtual work and solved analytically using Navier's method for simply supported plates. In contrast to previous works, the present formulation introduces a new trigonometric shear function that eliminates the need for shear correction factors, incorporates nonlocal scale effects relevant to nanoscale

structures, and provides a fully coupled hygrothermal model that simultaneously accounts for temperature and moisture gradients. Moreover, unlike numerical approaches or higher-order shear deformation models with numerous kinematic variables, the proposed Navier-type analytical solution achieves high accuracy with significantly reduced computational effort. The inclusion of Winkler and Pasternak foundation effects within a nonlocal hygrothermal framework, rarely addressed together in earlier studies, further distinguishes this work. Validation against benchmark TSDT solutions [36] confirms the accuracy of the proposed formulation, achieving precise results with significantly reduced computational effort. This comprehensive framework therefore provides a rigorous and efficient basis for analysing FG sandwich plates under combined mechanical, thermal, and hygroscopic loads and offers new insight into their behaviour in structurally demanding civil engineering environments.

2. Framework of the present theory

Accurate modelling of FGM sandwich plates under hygrothermal effects requires a formulation that simultaneously captures material gradation, environmental loading, and foundation interaction [37]. The mechanical response is strongly influenced by the through-thickness variation of elastic moduli, thermal expansion, and moisture coefficients, as well as by the coupling between bending, stretching, and shear deformation [38]. In the present work, these influences are incorporated within a continuum-based plate theory in which the kinematic field, constitutive relations, and hygrothermal strain components are consistently defined to reflect the graded nature of the material and the reactive support provided by the elastic foundation.

2.1. Kinematic and constitutive equations

Consider a rectangular functionally graded (FGM) sandwich plate of length a , width b , and total thickness h , defined in a Cartesian coordinate system (x, y, z) with the mid-plane at $z = 0$, as illustrated in Figure 1.

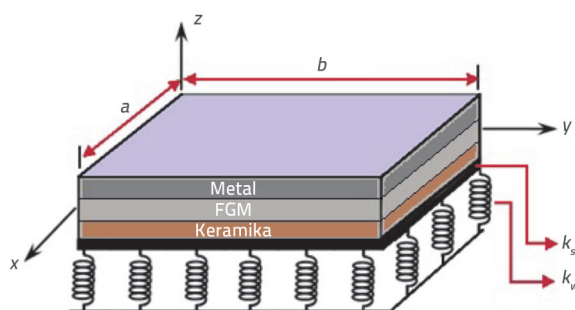


Figure 1. Geometry and material configuration of the metal-FGM-ceramic sandwich plate resting on a Winkler-Pasternak elastic foundation (FGM -functionally graded materials)

The plate consists of two homogeneous face sheets and a core made of functionally graded material, in which the properties vary continuously through the thickness according to a power-law distribution.

The variation of a generic material property $P_{(i)}(z)$, such as Young's modulus E , Poisson's ratio ν , the thermal expansion coefficient α , or the moisture expansion coefficient β , is expressed in Equation (1):

$$P_{(i)}(z) = P_m - P_{cm} V_{(i)}, \quad i = 1, 2, 3 \quad (1)$$

Equation 1 is obtained using the Voigt homogenisation scheme (rule of mixtures) [39, 40], which assumes an iso-strain condition through the thickness and perfect bonding between the ceramic and metal phases. The effective property $P_{(i)}(z)$ is therefore computed as a linear volume-fraction-weighted combination of the constituent phases.

Throughout the formulation, the subscript i denotes the layer index, where $i = 1$ refers to the top face sheet (metal), $i = 2$ to the FGM core, and $i = 3$ to the bottom face sheet (ceramic). The material properties $P_{(i)}(z)$ are defined for each layer according to its specific composition and thickness-wise distribution law.

In Equation (1), P_m and P_c are the property values of the metal-rich and ceramic-rich phases, respectively, where:

$$P_{cm} = P_m - P_c \quad (2)$$

The volume fraction distribution varies according to the type of layer. For the two homogeneous face sheets ($i = 1$ and $i = 3$), the material composition is constant through the thickness, with the volume fraction of the primary constituent equal to unity ($V_1 = 0$ for the top face sheet, $V_3 = 1$ for the bottom face sheet). For the FGM core layer ($i = 2$), the FG core ensures a smooth transition in mechanical and thermal properties, effectively mitigating interfacial stress concentrations. The material composition follows a continuous power-law distribution along the thickness direction z , expressed as:

$$\begin{cases} V_1 = 0 & za \quad -1/2 \leq \bar{z} \leq H_1 \\ V_2 = \left(\frac{\bar{z} - H_1}{H_2 - H_1} \right)^r & za \quad H_1 \leq \bar{z} \leq H_2 \\ V_3 = 1 & za \quad H_1 \leq \bar{z} \leq 1/2 \end{cases} \quad (3)$$

Here, the index r is a positive material gradient index ($r > 0$) that governs the variation of material properties across the plate thickness. The dimensionless coordinate $\bar{z}$ is defined as the ratio of the thickness coordinate z to the total plate thickness h ($\bar{z} = z/h$). Similarly, H_j denotes the normalised thickness of the j th layer, expressed as $H_j = h_j/h$, where h_j is the physical thickness of that layer.

The displacement field is defined according to the refined four-variable shear deformation theory, as given in Equation (4):

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial x} + k_1 f(z) \int \theta(x, y, t) dx \\ v(x, y, z, t) &= v_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial y} + k_2 f(z) \int \theta(x, y, t) dy \\ w(x, y, z, t) &= w_0(x, y, t) \end{aligned} \quad (4)$$

where u_0, v_0, w_0 are the mid-plane displacements, θ is the shear component of the transvers displacement, and $f(z)$ is a trigonometric shear function ensuring zero transverse shear stresses at $z = \pm h/2$.

The key novelty of the present formulation lies in the adoption of a newly proposed trigonometric transverse shear function, specifically designed to satisfy the essential physical requirement of vanishing transverse shear stresses at the top and bottom surfaces of the sandwich plate. This refinement ensures that the shear stress distribution is consistent with three-dimensional elasticity theory, thereby improving accuracy without the need for shear correction factors. In the proposed theory, the shape function $f(z)$ is defined as:

$$f(z) = \sin\left(\frac{2q\pi z}{h}\right) - \frac{2q\pi z}{h} \cos(qh); \quad g(z) = \frac{df(z)}{dz} \quad (5)$$

The proposed shape function has previously been applied to estimate the bending performance of FGM plates under patch loading [41]; however, to the best of the authors' knowledge, it has not yet been employed to analyse the response of FGMs under hygrothermal loading conditions within the framework of nonlocal elasticity theory.

In the adopted refined shear deformation theory, the parameter q appears in the thickness-dependent shape function $f(z)$ and controls the curvature of the transverse shear strain distribution across the plate thickness. Physically, q governs how rapidly the shear stresses vary from the mid-plane towards the top and bottom surfaces, ensuring that the shear stresses satisfy the stress-free boundary conditions at the external faces of the sandwich plate.

Lower values of q produce a smoother, more parabolic-like shear variation, while higher values make the shear distribution more localised near the interfaces [42]. Since the exact through-thickness shear distribution is generally unknown, several values of q (namely $q=5, 7$, and 9) were examined. This parametric study allows the determination of the value of q that yields results closest to previously published benchmark solutions for FG sandwich plates. The non-zero strain components are given by:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \varepsilon_{xy}^1 \end{Bmatrix} + f(z) \begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \varepsilon_{xy}^2 \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \quad (6)$$

Eringen's nonlocal elasticity theory extends the framework of classical (local) continuum mechanics by incorporating the effect

of long-range interatomic forces into the constitutive relations [43]. In this model, the stress at a given point in the material is not determined solely by the strain at that point but is also influenced by the strain field over a finite spatial region. This is achieved by introducing a material length-scale parameter into the constitutive equations, which captures size-dependent effects that become significant at the micro- and nanoscale [44]. According to Eringen's linear elasticity theory for sandwich nanoscale plates [45], the stress-strain relationships can be expressed in the following form [46]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} - \mu^2 \nabla^2 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha \Delta T - \beta \Delta C \\ \varepsilon_y - \alpha \Delta T - \beta \Delta C \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (7)$$

where ∇^2 represents the Laplacian operator, describing the sum of the second partial derivatives with respect to the in-plane coordinates x and y , and:

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}; \quad C_{12} = \frac{\nu E(z)}{1-\nu^2}; \quad C_{66} = C_{44} = C_{55} = \frac{E(z)}{2(1+\nu)} \quad (8)$$

The spatial distribution of temperature and moisture variations across the plate thickness is given by:

$$\Delta T = T - T_0; \quad \Delta C = C - C_0 \quad (9)$$

2.2. Governing equations of the hygrothermal-elastic sandwich system

To capture the coupled mechanical and hygrothermal behaviour of the FGM sandwich plate, the governing equations of motion are established using the principle of virtual work. This principle, which equates the internal strain energy to the external work performed by applied loads and environmental actions for all admissible virtual displacements, can be expressed analytically as:

$$\int_0^t (\delta U + \delta V - \delta K) dt = 0 \quad (10)$$

where δU denotes the variation of the strain energy, δK the variation of the kinetic energy, and δV the variation of the external work associated with the foundation reaction force density.

The variation of the strain energy for the FGM sandwich plate is expressed as:

$$\begin{aligned} \delta U &= \int_{-h/2}^{h/2} \int_A [\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz}] dA dz \\ &= \int_A [N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + \\ &\quad + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz} + S_{xz}^s \delta \gamma_{xz}] dA = 0 \end{aligned} \quad (11)$$

where A denotes the mid-plane area of the plate, and the stress resultants N , M , and S represent the in-plane forces, bending moments, and transverse shear forces, respectively, defined as:

$$\begin{Bmatrix} N_x & N_y & N_{xy} \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \quad (12)$$

$$(S_{xy}, S_{yz}) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{xy}) g(z) dz$$

The variation of the external work due to the reaction force density of the elastic foundation is expressed as:

$$\delta V = \int_{-h/2}^{h/2} \int_A f_e \delta w dA dz \quad (13)$$

where f_e denotes the reaction force density of the foundation acting against the transverse deflection.

$$f_e = K_w w(x, y) - K_s \nabla^2 w(x, y) \quad (14)$$

with k_w [N/m³] representing the Winkler modulus (normal stiffness) and k_s [N/m] the Pasternak shear coefficient, which accounts for transverse shear interactions in the foundation medium [47], where:

$$k_w = \frac{K_w a^4}{D_c}, \quad k_s = \frac{K_w a^2}{D_c}, \quad D_c = \frac{E_c h^3}{12(1-\nu^2)} \quad (15)$$

In this equation, K_w and K_s are the foundation stiffness parameters in the Pasternak model. Specifically, K_w represents the Winkler (normal) stiffness constant, accounting for the vertical reaction proportional to the local deflection $w(x, y)$, while K_s denotes the shear-layer stiffness constant, which models the foundation's shear interaction and acts on the Laplacian term $\nabla^2 w(x, y)$.

Performing integration by parts on Equation (10) yields the governing differential equations that describe the hygrothermoelastic response of the FGM sandwich plate:

$$\begin{cases} \delta u_0: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ \delta v_0: \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \\ \delta w_b: \frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} - K_w w_0 + K_s \nabla^2 w_0 = 0 \\ \delta \theta: -k_1 A \frac{\partial^2 M_x^s}{\partial x^2} - k_2 B \frac{\partial^2 M_y^s}{\partial y^2} - (k_1 A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}}{\partial x} + k_2 B' \frac{\partial S_{yz}}{\partial y} = 0 \end{cases} \quad (16)$$

Noting that:

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \quad (17)$$

In the present formulation, the applied temperature T and moisture concentration C are assumed to be constant over the (x, y) plane and to vary only along the thickness direction of the FG sandwich plate, as defined by:

$$K(z) = \Delta K \left(z + \frac{1}{2} \right)^p + k_b, \quad K = T, C \quad (18)$$

where $K = T, C$, with T representing the temperature effect and C the moisture effect.

By substituting Equation (7) into Equation (12), the stress resultants of the FGM sandwich plate are expressed in terms of the generalised strain components as [48]:

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{Bmatrix} [A] & [B] & [B^s] \\ [B] & [D] & [D^s] \\ [B^s] & [D^s] & [H^s] \end{Bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \varepsilon^{21} \\ \varepsilon^2 \end{Bmatrix} + \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix} + \begin{Bmatrix} N^C \\ M^{bC} \\ M^{sC} \end{Bmatrix} \quad (19)$$

where

$$\begin{Bmatrix} [A] & [B] & [B^s] \\ [B] & [D] & [D^s] \\ [B^s] & [D^s] & [H^s] \end{Bmatrix} = \begin{Bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & A_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & B_{11} & B_{12} & B_{11} & B_{12} & B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & B_{12} & B_{22} & B_{12} & B_{22} & B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{22}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{Bmatrix} \quad (20)$$

and

$$\begin{aligned} \varepsilon^0 &= \{\varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0\}, \varepsilon^1 = \{\varepsilon_x^1, \varepsilon_y^1, \varepsilon_{xy}^1\}, \varepsilon^2 = \{\varepsilon_x^2, \varepsilon_y^2, \varepsilon_{xy}^2\} \\ N &= \{N_{11}, N_{22}, N_{12}\}, M^b = \{M_{11}^b, M_{22}^b, M_{12}^b\}, M^s = \{M_{11}^s, M_{22}^s, M_{12}^s\} \\ N^k &= \{N_{11}^k, N_{22}^k, 0\}, M^{bk} = \{M_{11}^{bk}, M_{22}^{bk}, 0\}, M^{sk} = \{M_{11}^{sk}, M_{22}^{sk}, 0\}, k = T, C \end{aligned} \quad (21)$$

The corresponding stiffness components are defined as:

$$\begin{aligned} (A_{ij}, A_{ij}^s, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s) &= \int_{-h/2}^{h/2} (1, g(z)^2, z, f(z), z^2, z f(z), f(z)^2) C_{ij} dz \\ A_{ij}^s &= \int_{-h/2}^{h/2} C_{ij} [g(z)]^2 dz, \quad (i, j = 4, 5) \end{aligned} \quad (22)$$

For the hygrothermal stiffness components:

$$\begin{aligned} \{N_{11}^T, M_{11}^{bT}, M_{11}^{sT}\} &= -\sum_{i=1}^3 \int_{h_{i-1}}^{h_i} \Xi^{(i)} \Delta T \{1, z, f(z)\} dz, \\ \{N_{11}^C, M_{11}^{bC}, M_{11}^{sC}\} &= -\sum_{i=1}^3 \int_{h_{i-1}}^{h_i} \Upsilon^{(i)} \Delta C \{1, z, f(z)\} dz, \end{aligned} \quad (23)$$

Noting that:

$$N_{11}^k = N_{22}^k, \quad M_{11}^{bk} = M_{22}^{bk}, \quad M_{11}^{sk} = M_{22}^{sk}, \quad k = T, C \quad (24)$$

with

$$\Xi^{(i)} = \frac{E^{(i)}\alpha^{(i)}}{1-\nu^{(i)}}, \gamma^{(i)} = \frac{E^{(i)}\beta^{(i)}}{1-\nu^{(i)}}, \quad k = T, C \quad (25)$$

2.3. Proposed solution of the system

The analytical solutions for the hygrothermal dynamic response and buckling of simply supported FG sandwich plates are obtained using Navier's method. In this approach, the displacement components are expanded as double trigonometric (Fourier) series in terms of unknown coefficients, with the trigonometric functions chosen to inherently satisfy the simply supported boundary conditions. Accordingly, the displacements are expressed as:

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\mu x) \sin(\eta y) \\ V_{mn} \sin(\mu x) \cos(\eta y) \\ W_{mn} \sin(\mu x) \cos(\eta y) \\ X_{mn} \sin(\mu x) \cos(\eta y) \end{Bmatrix} \quad (26)$$

Here, U_{mn} , V_{mn} , W_{mn} and X_{mn} denote the unknown Fourier coefficients, while $\mu = m\pi/a$ and $\eta = n\pi/b$, with m and n representing the respective mode numbers in the x - and y -directions.

Although a detailed convergence study is beyond the scope of the present work, the numerical stability of the Navier-type double Fourier series was ensured by adopting a sufficiently large truncation number. It is well established in the literature that Navier solutions for plate and sandwich structures typically converge for $n \leq 60$ to 100 [49–52]. In this study, a truncation number of $n = 201$ was used for all calculations. Increasing the number of terms beyond this value does not produce any visible change in the structural response, confirming that the adopted truncation order provides numerically stable and fully converged results for the quantities of interest. Substituting Equation (14) into Eqs. (10) and (11) yields the equations of motion for the FG sandwich plate in terms of the unknowns U_{mn} , V_{mn} , W_{mn} and X_{mn} as follows:

$$\det \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} + k & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} = 0 \quad (27)$$

In Equation (27), "det" denotes the determinant of the stiffness matrix. Setting this determinant equal to zero ensures non-trivial solutions, corresponding to the characteristic equation used to determine the buckling loads of the FG sandwich plate.

3. Results and discussion

The proposed refined trigonometric shear deformation theory, coupled with Eringen's nonlocal elasticity, is applied to evaluate

the hygrothermal buckling behaviour of simply supported FGM sandwich nanoscale plates resting on Winkler–Pasternak elastic foundations. Unless otherwise specified, the plate configuration considered has a core-to-face thickness ratio $r = \rho = 1$, a ceramic–metal gradation in the FGM core governed by the power-law index n , and material properties listed in Table 1. All results are presented in non-dimensional form to facilitate comparison with benchmark solutions from the literature. Parametric studies investigate the influence of the gradient index, foundation stiffness parameters (k_w , k_s), shear function parameter q , slenderness ratio (a/h), and hygrothermal loading on the critical buckling temperature ΔT_{cr} . The performance of the proposed model is assessed against a third-order shear deformation theory (TSDT), demonstrating accuracy within 2 % across all investigated cases.

Table 1. Characteristics of the constituent materials of the FGM

	E [GPa]	ν	α_c ($1/^\circ\text{C}$)	β_c (wt % H_2O) ⁻¹
Metal (aluminum)	380	0.3	$7 \cdot 10^{-6}$	0.001
Ceramic (alumina)	70	0.3	$23 \cdot 10^{-6}$	0.44

In the following, the hygrothermal dynamic response and stability of simply supported FG sandwich plates are investigated using the newly developed refined four-variable shear deformation theory. This formulation employs a trigonometric transverse shear function that inherently satisfies zero-shear boundary conditions at the plate surfaces, thereby eliminating the need for empirical shear correction factors and improving the accuracy of coupled hygrothermal predictions. In the present study, hygrothermal effects are introduced following the formulation adopted in the reference work of Hellal et al. [36], where temperature and moisture fields are applied through prescribed differences across the plate thickness. The temperature field is defined by a uniform temperature rise ΔT applied relative to the reference temperature T_0 . For all numerical simulations, a temperature difference of $\Delta T = 50^\circ\text{C}$ is imposed, i.e., $T_b - T_0 = 50^\circ\text{C}$.

Moisture effects are introduced through the moisture concentration difference $\Delta C = C_b - C_0$. Two moisture cases are considered to represent low and high hygric loading conditions: $\Delta C = 0.1\%$ and 10% . These values correspond to small and significant humidity variations typically encountered in hygrothermally loaded multilayer composite plates [53–58].

The FG sandwich structure comprises alumina (ceramic) and aluminium (metal) layers, with material properties continuously graded through the thickness according to a prescribed volume fraction law, enabling a smooth transition in mechanical and thermal characteristics.

The non-dimensional critical thermal buckling parameter adopted in this analysis is defined as follows:

$$\Delta T_{cr} = 10^2 \alpha_c \Delta T \quad (28)$$

Table 2. Critical thermal buckling ΔT_{cr} of FG sandwich (I-I-I) square plate ($r = p = 1$, $\Delta C = 10\%$, $a = b$) in hygro-thermal environment

(k_w, k_s)	a/h	Critical thermal buckling ΔT_{cr}			
		q = 5	q = 7	q = 9	TSDT [24]
(0, 0)	4	5.4844	5.4941	5.4972	5.0828
	10	0.9444	0.9447	0.9448	0.9307
	15	0.4043	0.4044	0.4044	0.4016
	20	0.2123	0.2123	0.2123	0.2114
(0, 20)	4	21.4899	21.4996	21.5028	21.0884
	10	3.5053	3.5056	3.5057	3.4916
	15	1.5425	1.5426	1.5426	1.5397
	20	0.8525	0.8525	0.8525	0.8516
(20, 20)	4	22.3008	22.3105	22.3137	21.8992
	10	3.635	3.6353	3.6354	3.6214
	15	1.6002	1.6002	1.6003	1.5974
	20	0.8849	0.885	0.885	0.884
(100, 20)	4	25.5442	25.5539	25.5571	25.1427
	10	4.1539	4.1543	4.1544	4.1403
	15	1.8308	1.8309	1.8309	1.828
	20	1.0147	1.0147	1.0147	1.0138

To assess the accuracy of the proposed trigonometric shear function model of Kumar (2019) [41], a validation study is conducted using benchmark results from the higher-order third-order shear deformation theory (TSDT) presented by Hellal et al. (2019) [36]. The validation focuses on predicting transverse displacements and critical thermal buckling parameters of FG sandwich plates subjected to combined hygrothermal effects. Specifically, the test case involves the buckling of a sandwich plate resting on a Winkler–Pasternak elastic foundation under a linearly varying through-thickness thermal load. The predicted results are compared with those obtained from the TSDT approach [36], enabling a direct evaluation of the model's accuracy and reliability in thermal buckling analysis within a hygrothermal environment. As presented in Table 2, the numerical results obtained from the proposed refined shear deformation model show strong agreement with the benchmark solution of Hellal et al. (2019) [36] based on the third-order shear deformation theory (TSDT). The analysis considers the transverse deflection of FGM sandwich square plates under combined hygrothermal loading for different values of the trigonometric shear function parameter q . The comparison demonstrates that the present model reproduces the TSDT results with high fidelity, yielding relative errors consistently below 2 % for $q = 9$. This optimal convergence range confirms that the proposed trigonometric shear function accurately models the nonlinear distribution of transverse shear stresses through the thickness while capturing the coupling effects induced by temperature and moisture gradients. Such precision is critical for the reliable prediction of hygrothermal buckling behaviour in advanced FGM sandwich structures resting on elastic foundations.

In Figure 2, the graph illustrates the influence of the shear function parameter q on the critical thermal buckling temperature ΔT_{cr} of an (I-I-I) FG sandwich square plate ($r = p = 1$, $\Delta C = 10\%$, $a = b$) under combined hygrothermal loading. The parameter q , taken as an odd integer, governs the convergence behaviour of the proposed trigonometric shear deformation function. Results are presented for $q = 5, 7$, and 9 and are compared with the benchmark TSDT solution. Across all values of q , the proposed model predicts ΔT_{cr} values within approximately 6 to 7 % of the TSDT results, demonstrating consistent accuracy and robustness. This insensitivity to q variation within the chosen range confirms the stability of the proposed formulation and its reliability for practical hygrothermal buckling analysis of FG sandwich plates.

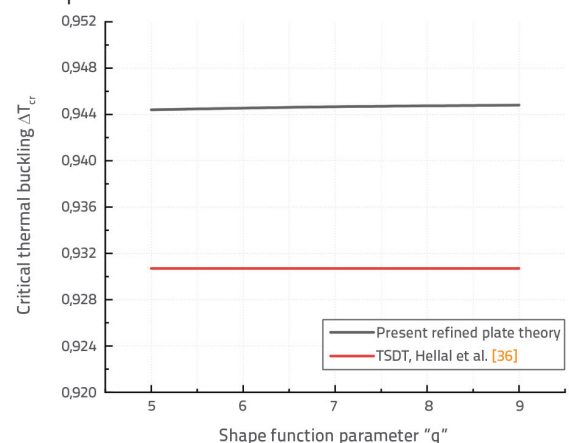


Figure 2. Effect of the parameter q in the proposed shape function on the estimated critical thermal buckling temperature ΔT_{cr} for a (1–1–1) FG sandwich square plate with ($r = p = 1$, $\Delta C = 10\%$, $a = b$)

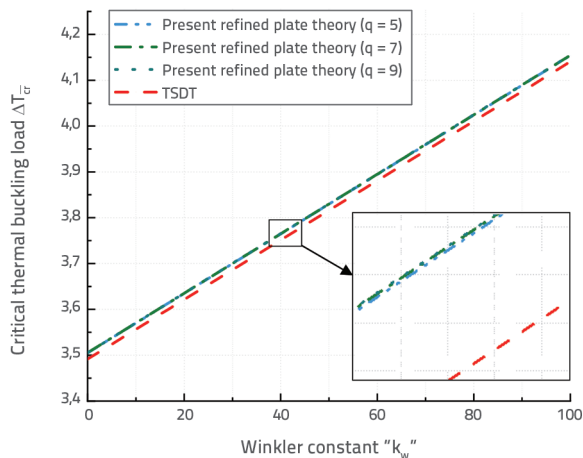


Figure 3. Effect of Winkler foundation stiffness constant (k_w) on the critical thermal buckling temperature (ΔT_{cr}) of a (I–I–I) FG sandwich square plate ($r = p = 1$, $\Delta C = 0.1\%$, $a = b$) under hygrothermal loading

Figure 3 presents the effect of the Winkler foundation stiffness constant (k_w) on the critical thermal buckling temperature (ΔT_{cr}) of an (I–I–I) FG sandwich square plate ($r = p = 1$, $\Delta C = 0.1\%$, $a = b$) subjected to hygrothermal loading. The results, as detailed

in Table 2, indicate that ΔT_{cr} increases monotonically with k_w , confirming the stabilising influence of the Winkler component of the elastic foundation. This trend is consistent across the range of slenderness ratios considered, with the proposed refined model maintaining close agreement with the TSDT benchmark and exhibiting percentage differences below 2 %. The findings highlight that incorporating realistic foundation stiffness into the analytical model significantly enhances buckling resistance, an effect of practical importance for the design of FGM sandwich panels in civil and mechanical engineering applications where substructure compliance cannot be neglected.

Figure 4 presents the variation of the critical thermal buckling temperature difference (ΔT_{cr}) for an FG sandwich square plate with $r = p = 1$ and $\Delta C = 0.1\%$, evaluated under various hygrothermal conditions. Results from the proposed refined shear deformation model ($q = 5, 7, 9$) are compared with the benchmark TSDT solution for different slenderness ratios (a/h) and foundation stiffness settings. The present model achieves excellent agreement, with percentage differences consistently below 2 % and a maximum deviation of 1.75 %, confirming its robustness across all geometric and environmental scenarios. This high level of accuracy, combined with reduced computational complexity, demonstrates the model's practical value for predicting thermal buckling in advanced FG sandwich structures supported by elastic foundations.

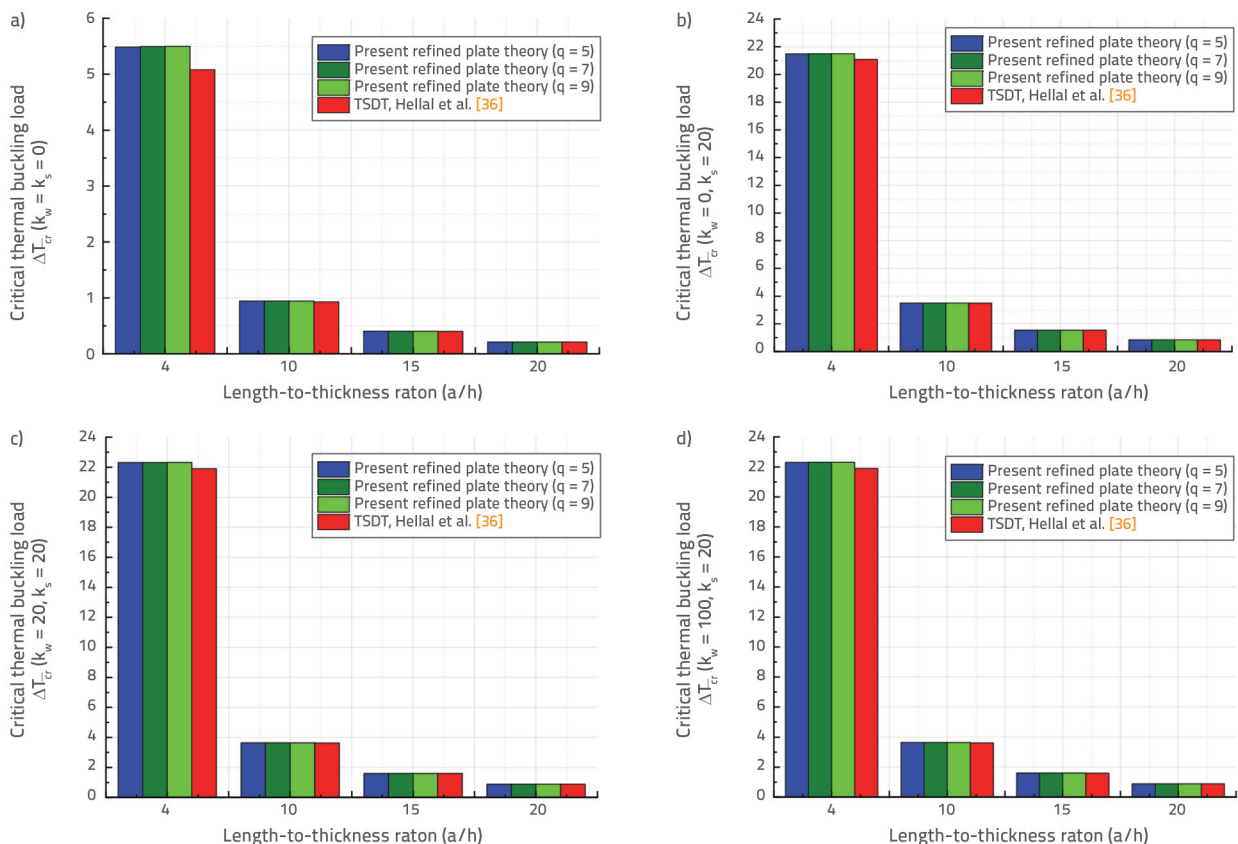


Figure 4. Histograms (a–d) showing the variation of the critical thermal buckling temperature (ΔT_{cr}) with slenderness ratio (a/h) for FG sandwich square plates under different foundation and shear stiffness conditions. Results from the present model are compared with those from the TSDT reference solution

Figure 5 illustrates the variation of the critical thermal buckling temperature ΔT_{cr} for a (1–1–1) FG sandwich nanoscale square plate with $\mu = 0.5$ as a function of the slenderness ratio a/h . The (1–1–1) configuration corresponds to equal thicknesses of the top face sheet, graded core, and bottom face sheet. The results reveal a monotonic decrease in ΔT_{cr} with increasing a/h , which is attributed to the reduction in bending stiffness as the plate becomes more slender. The proposed refined shear deformation theory closely matches the benchmark TSDT results of Hellal [36], showing excellent correlation across all slenderness ratios examined.

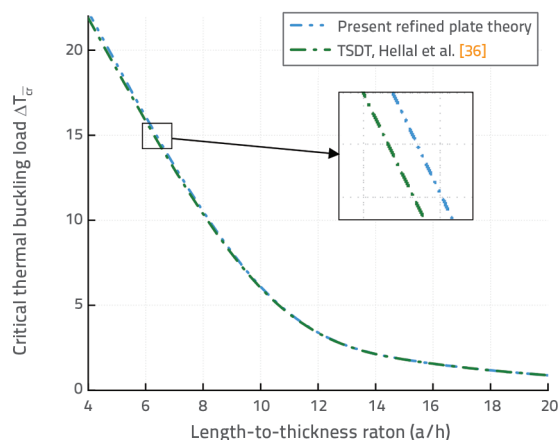


Figure 5. Variation of critical thermal buckling temperature ΔT_{cr} for a (1–1–1) FG sandwich nanoscale square plate with $\mu = 0.5$ as a function of the slenderness ratio a/h

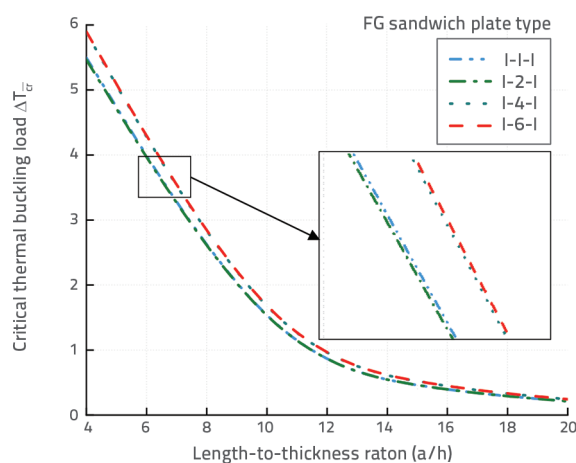


Figure 6. Critical thermal buckling of FG sandwich square plate without Winkler-Pasternak foundation against the thickness ratio (a/h), ($\mu = 2$, $r = p = 1$, $q = 9$, $\Delta C = 0.1\%$, $k_w = k_s = 0$)

Figure 6 illustrates the variation of the critical thermal buckling temperature (ΔT_{cr}) for FG sandwich nanoscale square plates with different face–core–face thickness configurations, evaluated for a foundation stiffness of $\mu = 2$, as a function of the slenderness ratio (a/h). The results showed that increasing the foundation stiffness enhanced the buckling resistance across all configurations, confirming the stabilising role of the elastic foundation. For each thickness configuration considered, ΔT_{cr} decreases consistently with increasing a/h due to the reduction in flexural rigidity for more slender plates. The close agreement between the predictions of the proposed refined model and benchmark TSDT solution demonstrates the capability of the formulation to accurately capture the coupled effects of geometric slenderness, sandwich configuration, foundation stiffness, and hygrothermal loading.

4. Conclusion

This study presented a refined four-variable shear deformation theory incorporating a novel trigonometric transverse shear function for the hygrothermal buckling and dynamic analysis of FGM sandwich nanoscale plates resting on Winkler–Pasternak elastic foundations. The proposed shear function inherently satisfied the zero-shear stress conditions at the plate surfaces, eliminating the need for empirical shear correction factors and ensuring compatibility with three-dimensional elasticity theory.

Validation against benchmark TSDT results demonstrated the high accuracy of the proposed model, with percentage differences generally below 2 % over a wide range of geometric slenderness ratios, foundation stiffness parameters, and face–core–face thickness configurations. Parametric studies revealed that increasing foundation stiffness significantly enhanced buckling resistance, whereas increasing the slenderness ratio reduced the critical thermal buckling temperature. The model also exhibited robustness with respect to variations in the shear function parameter q , maintaining prediction errors within 6 to 7 % compared with TSDT.

By combining computational efficiency with strong predictive capability, the proposed formulation provides a reliable analytical tool for the design and optimisation of advanced FGM sandwich structures in civil, marine, and aerospace engineering applications. Its ability to incorporate nonlocal elasticity effects, hygrothermal loading, and realistic support conditions makes it particularly valuable for modern infrastructure components exposed to coupled environmental and mechanical demands.

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